

On the Required Power of Centrifugal Pumps



December 3, 2017
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Introduction

Centrifugal pumps are representative of the most common plant equipment and are used for transporting various fluids.

The required power of the centrifugal pump is one of the important elements of pump selection, and calculation methods are shown in chemical engineering textbooks. Although these are simple formulas, they are often not well understood because they have different unit systems, or their assumptions are not clearly stated. Here, various equations for calculating the required power will be explained in terms of unit conversions.

Furthermore, by considering the premises of the required power equations, we will also explain what kind of calculations are required depending on the liquid transport conditions.



Required Power Indicated in Textbooks and Publications

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Kagaku Kougaku Binran, *in Japanese*: "Chemical Engineering Handbook" 3rd Ed. (1968)

2 · 7 · 2 所要動力

液体輸送に要する動力は、液体を高所に揚

げる場合にも、圧力容器に圧入する場合にも、あるいは甲に輸送官抵抗に)ちかつ物程を失えるだけの場合にも、目的のいかんにかかわらず、ポンプの全揚程 $H_t[\mathbf{m}]$ と流量 $Q[\mathbf{m}^3/\mathbf{sec}]$ とから、同様につぎの式で求められる。

水馬力= P_{th} = $\gamma QH_t/75$ =ポンプの理論所要動力

軸馬力= P_S = $100\,P_{th}/\eta$ =外部からポンプに加える動力

ポンプ効率 $\eta = 100 P_{th}/P_S$

ここに γ:液体の比重量 [Kg/m³]

P: 所要動力

[PS]

添字 th: 理論馬力, S: 軸馬力

2.7.2 Required Power

The required power for liquid transport, regardless of the purpose, whether the liquid is raised to a high place, fed to a pressurized vessel, or simply given a head to overcome transport resistance, is similarly determined by the following equation from the total head of the pump H_t [m] and the flow rate Q [m³/sec].

Water Horsepower = $P_{th} = \gamma Q H_t / 75$ = theoretical required power of pump

Shaft Horsepower = $P_S = 100 P_{th}/\eta = \text{power to change the pump from the outside}$

Pump Efficiency = $= 100 P_{tb}/P_{S}$

where γ: Specific Weight of Liquid [Kg/m³]

P: Required Power [PS]

Subscripts th: Theoretical Horsepower, S: Shaft Horsepower

"Kagaku Purosesu no Hyoukahou", *in Japanese*: "Chemical Processes Evaluation Methods" (1973, Maruzen p96)

4.6.2 軸 馬 力

ボンブの機略コストは、所要動力の関数として表わされている場合が多く、 またプロセスのユーティリティー使用量の計算にも、各ポンプの軸馬力を計算 する必要がある。ポンプの軸馬力は、式 (4·54) により計算できる。

$$S = \frac{0.22 \times \rho_L \times Q \times H}{\eta} \tag{4.54}$$

上式において、S: ボンプの軸馬力 [\mathbb{P}]、 ρ_L : 液の比重、Q: 液流量 [\mathbb{m} ³ / \mathbb{m} 1]、H: 揚程 [\mathbb{m}]、 η : ボンプ効率 (近似的には 0.5 とする)、である.

 ρ_L :比重量 $\left[\frac{gf}{cm^3}\right]$

4.6.2 Shaft Horsepower

The approximate cost of the pump is often expressed as a function of the required power, and the calculation of the utility usage of the process also requires the shaft horsepower of each pump to be calculated. The shaft horsepower of the pump can be calculated by Eq. (4.54).

$$S = \frac{0.22 \times \rho_L \times Q \times H}{\eta} \tag{4.54}$$

In the above equation, S: Pump Shaft Horsepower [HP], ρ_L : Specific Gravity of Fluid[gf/cm³], Q: Fluid Flow Rate [m³/min], H: Head [m] η : Pump Efficiency (approximately 0.5)

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"Kagaku Purosesu no Keisanhou", *in Japanese*: "Chemical Engineering Calculation Methods" (Denki University Press, 1999, p219)

図 17-1 に示すように実際に揚水する高さを実揚程というが、輸送管内の摩擦 損失"などの損失があるので、それらの損失に相当する損失水頭(吐出し管損失 水頭・吸込管損失水頭)を加えたものを輸送物質に与えなければならない。ポン プが水に与えなければならない損失水頭を全揚程といい、次式で表される。

全揚程 H(m)= 実揚程 $H_a(m)$ + 損失水頭 $H_l(m)$ ············(17-1)

実揚程=吐出し実揚程+吸込実揚程………(17-2)

ただし、吸込実揚程は7mが限界とされている。

比重量 $\gamma(kgf/m^3)$, 吐出し量 $Q(m^3/s)$ の水を全揚程 H(m) の高さに揚水する

場合、ポンプは $\gamma QH(\text{kgf·m/s})$ の仕事を しなければならないから、それに必要な動 力 L_W は次のようになる。

 $L_W = \gamma QH/102$ [kW](17-3) γ :水の比重量=1 000 kgf/m³, Q: 吐出し量[m³/s], H: 全揚程[m]

実際には、ポンプ内の流体摩擦や軸受などの摩擦抵抗による損失があるから、ポンプの軸に与える動力の軸動力 L は、ポンプの水動力 L_W より大きくなくてはならない。ゆえにポンプの効率 η は次式で表される η 。

 $\eta = L_W/L$ ·········(17-4)

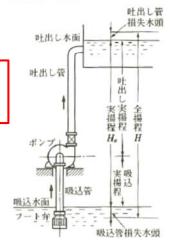


図 17-1 ポンプ揚水装置



As shown in Fig. 17-1, the actual pumping height is called the actual head, but since there are losses such as friction losses¹⁾ in the transport piping, the transport material must be provided with an added water head loss corresponding to those losses (discharge pipe water head loss, suction pipe water head loss). The water head loss that the pump has to give to water is called the total head, and is expressed by the following equations.

Total Head H [m] = Actual Head H_a [m] + Water Head Loss H_t [m] ... (17-1) Actual Head = Actual Discharge Head + Actual Suction Head (17-2) However, the actual suction head is limited to 7 m.

When pumping water with a specific weight γ [kgf/m³] and a discharge rate of Q [m³/s] to the height of the total head H [m], since the pump must do the work γQH [kgf.m/s], the required power L_w is as follows.

$$L_W = \gamma QH/102 \text{ [kW]}$$
 (17-3)
 γ : Specific Gravity of Water = 1000 kgf/m³,
 Q : Discharge flow Rate [m³/s], H : Total Head [m]

In practice, since there is a loss due to fluid friction in the pump or frictional resistance such as from shaft bearings, the shaft power L given to the pump shaft must be larger than the water power L_W of the pump. Therefore, the efficiency of the pump is expressed by the following equation²⁾.

$$\eta = L_W/L$$
(17-4)

Although there may be other notations, the formulations of the equations (especially the coefficients) will differ depending on the unit system used and are difficult to apply.

Now that they have been standardized to the SI unit system, we will explain these formulas by comparing with SI units.

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2. Required Power in SI Unit System "Kagaku Kougaku Binran", *in Japanese*: "Chemical Engineering Handbook", 7th Ed., (2011)



"Kagaku Kougaku Binran", in Japanese: "Chemical Engineering Handbook", 7th Ed., (2011)

The power determined from the total head H given to the liquid and the flow rate Q is called the theoretical power or water power, and a formula is given in p206.

Water power P_{th} [W] = ρgQH = Theoretical Power (3.333)

 ρ : Density

g: Gravitational Acceleration 9.807 [$\frac{m}{s^2}$]

Q: Flow Rate $\left[\frac{\text{m}^3}{\text{s}}\right]$

H: Total Head [m]

Check the units of the formula.

$$[\rho gQH] = \frac{kg}{m^3} \frac{m}{s^2} \frac{m^3}{s} m = \frac{kg \frac{m^2}{s^2}}{s}$$
$$= \frac{[Kinetic Energy]}{s} = \frac{J}{s} = W$$

It can be seen that the unit of Eq. (3.333) is [W].

It is necessary to understand Bernoulli's equation to clarify the meaning of *H* (total head).

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Bernoulli's Equation and Units

Bernoulli's equation is a conservation law of mechanical energy, as shown in Eq. (3.26) on p.138 ("Kagaku Kougaku Binran", *in Japanese*: "Chemical Engineering Handbook", 7th Ed.).

$$\frac{1}{2}u^2 + gz + \frac{P}{\rho} = \text{Constant} \ (3.26)$$

 $u : \text{Flow Velocity} \left[\frac{\mathsf{m}}{\mathsf{s}} \right]$

g: Gravitational Acceleration 9.807 [$\frac{m}{s^2}$]

z: Height [m]

$$P: \text{Pressure } [\text{Pa}] = \left\lceil \frac{N}{m^2} \right\rceil = \left\lceil \frac{\text{kg}}{\text{m} \cdot \text{s}} \right\rceil$$

 ρ : Density $\left\lceil \frac{\mathrm{kg}}{\mathrm{m}^3} \right\rceil$

Check the units of the formula.

$$\left[\frac{1}{2}u^{2}\right] = \frac{m^{2}}{s^{2}} = \frac{kg \frac{m^{2}}{s^{2}}}{kg} = \frac{[Kinetic Energy]}{kg} = \frac{J}{kg}$$
$$[gz] = \frac{m}{s^{2}} m = \frac{m^{2}}{s^{2}} = \frac{[Potential Energy]}{kg} = \frac{J}{kg}$$
$$\left[\frac{P}{\rho}\right] = \frac{kg}{ms^{2}} \frac{m^{3}}{kg} = \frac{m^{2}}{s^{2}} = \frac{[Pressure Energy]}{kg} = \frac{J}{kg}$$

It can be seen that the unit of Eq. (3.26) are [J/kg].



Application of Bernoulli's Equation to Pump Energy

In order to calculate the energy required to pump from point 1 to point 2 using Bernoulli's equation, Eq. (3.27) is shown in p139 ("Kagaku Kougaku Binran", in Japanese: "Chemical Engineering Handbook", 7th Ed.). In turbulent flow, Eq. (3.27a) is obtained.

$$\frac{1}{2}u^{2} + gz + \frac{P}{\rho} = \text{constant}$$
 (3.26)
$$\frac{1}{2}\alpha u_{1}^{2} + gz_{1} + \frac{P_{1}}{\rho} + W = \frac{1}{2}\alpha u_{2}^{2} + gz_{2} + \frac{P_{2}}{\rho} + \sum_{i} F_{i}$$
 (3.27)

Since for turbulent flow, a = 1 (p139, line 8)

$$\frac{1}{2}u_1^2 + gz_1 + \frac{P_1}{\rho} + W = \frac{1}{2}u_2^2 + gz_2 + \frac{P_2}{\rho} + \sum_i F_i \qquad (3.27a)$$

W: Energy the pump applies [J/kg]

 F_i : Sum of energy loss per unit mass of fluid [J/kg]

$$F_i = \frac{\Delta p_i}{\rho} = 4f \frac{L}{d} \frac{1}{2} u^2$$
 Fanning equation (3.29)

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Transformation of Bernoulli's Equation to Express the Head

$$\left[\frac{J}{kg}\right]... \frac{1}{2}u_1^2 + gz_1 + \frac{P_1}{\rho} + W = \frac{1}{2}u_2^2 + gz_2 + \frac{P_2}{\rho} + \sum_i F_i \quad (3.27a)$$

Dividing both sides by gravitational acceleration,

$$\left[\frac{J}{kg}\frac{s^2}{m}\right] = [m]... \frac{1}{2g}u_1^2 + z_1 + \frac{P_1}{\rho g} + \frac{W}{g} = \frac{1}{2g}u_2^2 + z_2 + \frac{P_2}{\rho g} + \frac{\sum_{i} F_i}{g}$$
(3.27b)

[m]...
$$\frac{W}{g} = \frac{1}{2g} (u_2^2 - u_1^2) + (z_2 - z_1) + \frac{P_2 - P_1}{\rho g} + \frac{\sum_{i} F_i}{g}$$
 (3.27c)

When the suction side and the discharge side pipe diameter of the pump are the same, since $u_1 = u_2$,

[m]...
$$\frac{W}{g} = (z_2 - z_1) + \frac{P_2 - P_1}{\rho g} + \frac{\sum_{i} F_i}{g}$$
 (3.27d)

Each term of (3.27c) and (3.27d) is called a head.

 $z_2 - z_1$: Actual head

 $P_2 - P_1$: Pump head due to the pressure difference between the source and destination tanks

: Head due to friction losses of fluid

The total head equation is given on p205.

$$H = \frac{P_1 - P_2}{\rho g} + h_a + h_{ls} + h_{ls}$$
 (3.330)

$$h_a = z_1 - z_2$$
 (Actual head)

$$h_a = z_1 - z_2 \qquad \text{(Actual head)}$$

$$h_{ls} + h_{ls} = \frac{\sum_{i} F_i}{g} \qquad \text{(Head loss)}$$



Often used Approximations for Head Calculations

[m]...
$$\frac{W}{g} = \frac{1}{2g} (u_2^2 - u_1^2) + (z_2 - z_1) + \frac{P_2 - P_1}{\rho g} + \frac{\sum_{i} F_i}{g}$$
 (3.27c)

When the suction side and the discharge side pipe diameter of the pump are the same, since $u_1 = u_2$,

[m]...
$$\frac{W}{g} = (z_2 - z_1) + \frac{P_2 - P_1}{\rho g} + \frac{\sum_{i} F_i}{g}$$
 (3.27d)

Eq. (3.27f) is sometimes called the theoretical required power calculation equation of a pump.

When the pipe diameter is constant and the pressure of the liquid feed source tank and the pressure of the liquid feed destination tank are the same, $P_1 = P_2$

[m]...
$$\frac{W}{g} = (z_2 - z_1) + \frac{\sum_{i} F_i}{g}$$
 (3.27e)

In the case of in-plant liquid transportation of a few decameters, since the actual head is dominant

[m]...
$$\frac{W}{g} = (z_2 - z_1)$$
 (3.27f)

For water intake pumps and pipelines where horizontal distance is more important than the level differences

[m]...
$$\frac{W}{g} = \frac{\sum_{i} F_{i}}{g}$$
 (3.27g)

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Theoretical Required Power and Head

[m]...
$$\frac{W}{g} = \frac{1}{2g} (u_2^2 - u_1^2) + (z_2 - z_1) + \frac{P_2 - P_1}{\rho g} + \frac{\sum_{i} F_i}{g}$$
 (3.27c)

When the head is defined by H [m], the above equation is as follows.

[m]...
$$\frac{W}{g} = H = \frac{1}{2g} (u_2^2 - u_1^2) + (z_2 - z_1) + \frac{P_2 - P_1}{\rho g} + \frac{\sum_{i} F_i}{g}$$

Multiplying both sides by $\rho g Q$ (Q: flow rate [m³/s])

$$\rho g Q \frac{W}{g} = \rho g Q H \qquad \therefore \rho Q W = \rho g Q H$$

If we verify the physical units

$$[\rho QW] = \frac{kg}{m^3} \frac{m^3}{s} \frac{J}{kg} = \frac{J}{s} = W$$

$$[\rho gQH] = \frac{kg}{m^3} \frac{m}{s^2} \frac{m^3}{s} m = \frac{kg}{s} \frac{m^2}{s} = \frac{J}{s} = W$$

The theoretical required power expressed by the head is $w = \rho gQH$ [W].



3. Verification of Required Power shown in Textbooks and Publications

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SI Unit System and Engineering Unit System

Various unit systems have been used before physical units were unified into the SI unit system.

One important unit among them is "force", which is represented by [N] in the SI unit system. For example, the force (weight) felt when holding a mass of 1 kg on the surface of Earth is: force = mass × acceleration

$$F = 1 \text{ [kg]} \times 9.8 \text{ [m/s}^2\text{]} = 9.8 \text{ [N]}.$$

However, before using the SI unit system, since it was convenient in daily life, a weight of a mass of 1 kg called 1 kilogram force was used instead of [N], using the notations 1 [kgf] or 1 [Kg] (note that the K is capitalized).

A unit system having a force in kgf or Kg is called an engineering unit system, and was a typical unit system used in textbooks and plants until the 1980s. Nowadays, there are fewer opportunities to directly use engineering unit systems because the SI system is the representative system of units. However, since engineering unit systems were used during the period when chemical engineering was most actively researched, they are still important when referring to many specialized books that were written at the time.

Therefore, it is desirable to understand engineering unit systems when using literary references of the past and the like.

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Gravity Conversion Coefficient in Engineering Unit System

Force = mass \times acceleration $(f = m \times a)$

$$f = 1 \text{kg} \times 9.807 \frac{\text{m}}{\text{s}^2} = 9.807 \frac{\text{kgm}}{\text{s}^2} = 9.807 \text{N}$$

A gravity conversion coefficient g_c is defined to convert

$$9.807 \frac{\text{kgm}}{\text{s}^2} (\text{N}) \text{ to 1 kg.}$$

$$g_C = 9.807 \frac{\text{kg} \times \text{m}}{\text{kgf} \times \text{s}^2}$$

$$1 \text{kgf} \times g_C = 1 \text{kgf} \times 9.807 \frac{\text{kg} \times \text{m}}{\text{kgf} \times \text{s}^2} = 9.807 \frac{\text{kgm}}{\text{s}^2} \text{ (N)}$$

The law of Newton expressed in the units of kgf is

$$w[kgf] = \frac{m[kg] \times g[\frac{m}{s^2}]}{g_C[\frac{kg \times m}{kgf \times s^2}]} = m \frac{g}{g_C} [kgf]$$

The numbers *w* and *m* match.

Example

Mass	Force
1kg	$1 \text{kg} \times \frac{\text{g}}{\text{g}_{\text{C}}}$

Density

Specific Weight

$$1 \frac{kg}{m^3} \qquad 1 \frac{kg}{m^3} \times \frac{g}{g_C}$$

$$= 1 \frac{kg}{m^3} \frac{m}{s^2} \frac{kgfs^2}{kgm} = 1 \frac{kgf}{m^3}$$

$$\rho \frac{kg}{m^3} \qquad \rho \frac{kg}{m^3} \times \frac{g}{g_C}$$

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As shown in Fig. 17-1, the actual pumping height is called the actual head, but since there are losses such as friction losses ¹³ in the transpor piping, the transport material must be provided with an added water head loss corresponding to those losses (discharge pipe water head loss, suction pipe water head loss). The water head loss that the pump has to give to water is called the total head, and is expressed by the following equations.

When pumping water with a specific weight γ [kgf/m²] and a discharge rate of \mathbb{Q} [m³/s] to the height of the total head \mathbb{H} [m], since the pump must do the work γQH [kgf/m/s], the required power Lw is a follows. $I_H = \gamma QH/102$ [kW] (17-3)

γ: Specific Gravity of Water = 1000 kgf/m³, Q: Discharge flow Rate [m³/s], H: Total Head [m]

In practice, since there is a loss due to fluid friction in the pump or frictional resistance such as from shaft bearings, the shaft power L given to the pump haft must be larger than the water power L_{IT} of the pump. Therefore, the efficiency of the pump is expressed by the following equation: $\eta^{=}L_{IT}(L_{IT}) = \frac{1}{2} \left(\frac{1}{2$

$$L_W = \frac{\gamma QH}{102} [\text{kW}]$$

 γ : Specific Weight [$\frac{\text{kgf}}{\text{m}^3}$]

Q: Flow Rate $\left[\frac{\text{m}^3}{\text{s}}\right]$

H: Total Head [m]

Although there may be other notations, the formulations of the equations (especially the coefficients) will differ depending on the unit system used and are

Theoretical Required Power $w = \rho gQH$ [W]

Dividing both sides by g_c , $\frac{w}{w} = \rho \frac{g}{Q} OH$

Mifficult to apply.

Now that they have been standardized to the SI unit system, we will explain these formulas by comparing with SI

 $\frac{w}{g_C} = \rho \frac{g}{g_C} QH$ $\frac{w}{g_C} = \gamma QH \quad (\because \rho \frac{g}{g_C} = \gamma)$ $w = g_C \gamma QH \quad [W]$ $L_W = \frac{g_C}{1000} \gamma QH \quad [kW]$ $= \frac{9.807}{1000} \gamma QH = \boxed{\frac{\gamma QH}{102.0} \quad [kW]}$

Thinking in the SI unit system,



4.6.2 Shaft Horsepower The approximate cost of the pump is often expressed as a function of the required power, and the calculation of the utility usage of the process also requires the shaft horsepower of each pump to be calculated. The shaft horsepower of the pump can be calculated by Eq. (4.54).

$$S = \frac{0.22 \times \rho_L \times Q \times H}{n}$$
 (4.54)

In the above equation, S: Pump Shaft Horsepower [HP], ρ_L : Specific Gravity of Fluid[gf/cm³], Q:

$$S = \frac{0.22 \rho_L QH}{\eta}$$

Theoretical Required Power

$$S' = 0.22 \rho_L QH$$

$$S, S'$$
: Power [HP]

$$ho_{\!\scriptscriptstyle L}$$
 : Specific Weight [$rac{{
m gf}}{{
m cm}^3}$]

$$Q$$
: Flow Rate $\left[\frac{\text{m}^3}{\text{min}}\right]$

$$\eta$$
: Efficiency [-]

Thinking in the SI unit system,

Theoretical Required Power $w = \rho g g H$ [W]

$$(q : Flow Rate [m^3/s])$$

Dividing both sides by g_c ,

$$\frac{w}{g_C} = \rho \frac{g}{g_C} qH = \gamma qH \quad (:: \rho \frac{g}{g_C} = \gamma)$$

$$= 1000 \rho_L qH$$

$$(Q\rho_L[\frac{\mathrm{gf}}{\mathrm{cm}^3}], \gamma[\frac{\mathrm{kgf}}{\mathrm{m}^3}], \gamma=1000\rho_L)$$

$$w = 1000 g_C \rho_L \frac{Q}{60} H$$
 [W]

$$= \frac{1000g_C}{1000 \times 60} \rho_L QH \quad [kW]$$

Since
$$1[HP] = 0.7457[kW]$$

$$S' = \frac{1}{0.7457} \frac{1000 \times 9.807}{1000 \times 60} \rho_L QH \text{ [HP]}$$

$$= 0.219 \rho_L QH \text{ [HP]} \cong 0.22 \rho_L QH \text{ [HP]}$$

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The required power for liquid transport, regardless of the purpose, whether the liquid is raised to a high place, fed to a pressurized vessel, or simply given a head to overcome transport resistance, is similarly determined by the following equation from the total head of the pump $H_{\Gamma}[m]$ and the flow rate Q [m3/sec].

Water Horsepower = $P_{th} = \gamma Q H_t / 75$ = theoretical required power of pump Shaft Horsepower = $P_S \neq 100 \ P_{th} / \eta$ = power to change the pump from the or Pump Efficiency = $= 100 \ P_{th} / P_S$

Printp Linderby — — 00 2 m² 2

P: Required Power [PS]

Subscripts th: Theoretical Horsepower, S: Shaft Horsepower

$$P_{th} = \frac{\gamma Q H_t}{75}$$

$$P_{th}$$
: Power [PS]

$$\gamma$$
: Specific Weight [$\frac{\text{kgf}}{\text{m}^3}$]

$$Q: \text{Flow Rate } \left[\frac{\text{m}^3}{\text{s}} \right]$$

$$H_t$$
:Total Head [m]

Thinking in the SI unit system,

Theoretical Required Power $w = \rho gOH$ [W] Dividing both sides by g_c ,

$$\frac{w}{g_C} = \rho \frac{g}{g_C} QH$$

$$\frac{w}{g_C} = \gamma Q H \quad (\because \rho \frac{g}{g_C} = \gamma)$$

$$w = g_C \gamma Q H$$
 [W]

$$= \frac{g_C}{1000} \gamma Q H \text{ [kW]}$$

Since
$$1[HP] = 0.7355 [kW]$$

$$P_{th} = \frac{1}{0.7355} \frac{g_C}{1000} \gamma QH = \frac{\gamma QH}{75.00} \text{ [PS]}$$



Conclusion

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- Although various units are used for pump theoretical power calculation equations, it can be understood that the same equations can be obtained from Bernoulli's equation in the SI unit system.
- The theoretical required power is almost determined by the actual head (i.e., the difference in [m] between the liquid level of the source and destination tanks), when assuming that the suction and discharge pipe diameter of the pump is the same, that the pressure of the liquid feed source and liquid feed destination tank is the same, and that near-field transportation is performed.
- Books written before SI units were used often contain detailed descriptions and many points that can be helpful, and it is useful to become familiar with the gravity conversion coefficient to understand these.